Chap 02 Basic Laws
Outline

• Introduction
• Ohm’s Law
• Nodes, Branches, and Loops
• Kirchhoff’s Laws
• Series Resistors and Voltage Division
• Parallel Resistors and Current Division
• Wye-Delta Transformations
Introduction

• Study fundamental laws that govern electric circuits.
  – Ohm’s law
  – Kirchhoff’s current law
  – Kirchhoff’s voltage law
Electrical Resistance

- **Resistance** is the capacity of materials to impede the flow of current (or electric charge).
- A circuit element that displays such resistive behavior is called a resistor.

\[
R = \rho \frac{l}{A}
\]

- \(\rho\) = resistivity in ohm-meters (\(\Omega \cdot m\))
- \(l\) = length of material (m)
- \(A\) = cross-sectional area (m\(^2\))
- \(R\) = resistance in ohms (\(\Omega\))
(a) Resistor,
(b) Circuit symbol for resistance.

Cross-sectional area $A$

Material with resistivity $\rho$

Electrical Resistance (cont.)
Categories of Materials

- Materials can be categorized into three main groups regarding their electrical conduction properties:
  - Insulators
  - Conductors
  - Semiconductors
Georg Simon Ohm (1787-1854)
Ohm’s Law

- **Ohm’s Law:** The voltage \( v \) across a resistor is directly proportional to the current \( i \) flowing through the resistor.

- Ohm’s law provides an algebraic relationship between voltage and current for a resistor,

\[
v = iR
\]

\( v = \) voltage in volts \( (V) \),
\( i = \) current in amperes \( (A) \)
\( R = \) resistance in ohms \( (\Omega) \)
Linear/Nonlinear Resistors

- The resistance $R$ (measured in ohms) of an element denotes its ability to resist the flow of electric current.
- We use ideal (linear) resistors in this course that we assume is constant in value and that do not vary over time (time invariant).
The $i$-$v$ characteristic of:
(a) A linear resistor,
(b) A nonlinear resistor.

(a) Slope = $R$

(b) Slope = $R$
Further Topics Related to Ohm’s Law

• A **short circuit** is a circuit element with resistance approaching zero \((R \to 0)\),
  \[ \nu = iR = 0. \]

• An **open circuit** is a circuit element with resistance approaching infinity \((R \to \infty)\),
  \[ i = \lim_{R \to \infty} \frac{\nu}{R} = 0 \]
Further Topics Related to Ohm’s Law (cont.)

(a) Short circuit ($R = 0$),
(b) Open circuit ($R = \infty$)
Examples of Fixed Resistors

Fixed resistors:
(a) wirewound type.
(b) carbon film type.
Example of Variable Resistors

Variable resistors:
(a) Composition type
(b) Slider pot.
Circuit Symbols of Variable Resistors

Circuit symbol for:
(a) A variable resistor in general,
(b) A potentiometer or a pot.
Resistors in a thick-film circuit.
Conductance

- **Conductance**, measured in siemens (S)/ mho (℧), is the ability of an element to conduct electric current,

$$G = \frac{1}{R} = \frac{i}{v}$$

$v$ = voltage in volts (V)

$i$ = current in amperes (A)

$R$ = resistance in ohms (Ω)
• Remember that power can be expressed as

\[ p = vi \]

- \( p \) = power in watts (W)
- \( v \) = voltage in volts (V)
- \( i \) = current in amperes (A)
• Using Ohm’s law, power can also be expressed in terms of the current and the resistance as

\[ p = vi = (iR)i = i^2R \]

• Power can also be expressed in terms of voltage and resistance as

\[ p = \frac{v^2}{R} \]
Power at the Terminals of a Resistor (cont.)

- Regardless of voltage polarity and current direction, power at the terminals of a resistor is positive ($p > 0$), i.e. a resistor absorbs power from the circuit.

- The consumed energy in resistor will be dissipated in terms of heat to the environment. A heater is a typical resistor.

- If the power supplied to the resistor is so large that the energy consumed in the resistor cannot reach the thermal equilibrium, the resistor will be broken; hence each resistor has a power limit.
Example 2.2

- In the circuit shown in the right figure, calculate the current $i$, the conductance $G$, and the power $p$.

\[ i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA} \]

\[ G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS} \]

\[ p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW} \]

or

\[ p = i^2R = (6 \times 10^{-3})^2 \times 10^3 = 180 \text{ mW} \]

or

\[ p = v^2G = (30)^20.2 \times 10^{-3} = 180 \text{ mW} \]
Nodes, Branches and Loops

• **A network:** An interconnection of elements of devices.

• **A circuit:** A network provides one or more closed paths.

• A **branch** represents a single element such as a voltage source or a resistor.

• A **node** is the point of connection between two or more branches.

• A **loop** is any closed path in a circuit.
A Three-node Circuit

3 Nodes, 5 branches and 3 independent loops.
The three-node circuit of Fig. 2.10 is redrawn.

![Diagram of a three-node circuit](image)
**Independent Loop:** A loop contains at least one branch which is not a part of any other independent loop.

**Fundamental Theorem of Network Topology:** A network with $b$ branches, $n$ nodes and $l$ independent loop satisfy the following equation.

\[ b = l + n - 1 \]
Series and Parallel Connections

• Two or more elements are in **series** if they exclusively share a single node and consequently carry the **same current**.

• Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the **same voltage** across them.
An Example of Series and Parallel Connections

- **Series connection**: 10-V voltage source and 5-Ω resistor.
- **Parallel connection**: 2-A current source and 6-Ω resistor.
Gustav Robert Kirchhoff (1824-1887)
Kirchhoff’s Current Law

- **Kirchhoff’s current law (KCL):** The algebraic sum of currents entering a node (or a closed boundary) in a circuit equals zero.

\[
\sum_{n=1}^{N} i_n = 0
\]

- \( N \) = number of branches connected to the node
- \( i_n \) = \( n \)th current entering (or leaving) the node
Kirchhoff’s Current Law (cont.)

- Remember that a node is a point where two or more circuit elements meet.
- To use KCL, an algebraic sign corresponding to a reference direction must be assigned to every current at the node, e.g. a positive sign for currents leaving a node requires a negative sign for currents entering a node.
Proof of Kirchhoff’s Current Law

• Assume a set of currents $i_k(t)$, $k=1,2,…$, flow into a node. Their algebraic sum is

$$i_T(t) = i_1(t) + i_2(t) + …$$

• Integrating both sides of the above equation gives

$$q_T(t) = q_1(t) + q_2(t) + …$$

• By the Law of conservation of electric charge

⇒ The algebraic sum of electric charges at the node must not change; that is, the node store no net charge.

Hence

$$q_T(t) = 0 \rightarrow i_T(t) = 0.$$
Currents at a Node Illustrating KCL

\[ i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0 \]

\[ i_1 + i_3 + i_4 = i_2 + i_5 \]
Applying KCL to a Closed Boundary
A Simple Application of KCL

Combine current sources in parallel
(a) The original circuit.
(b) The equivalent circuit.

\[ I_T + I_2 = I_1 + I_3 \]
\[ \Rightarrow I_T = I_1 - I_2 + I_3 \]
Kirchhoff’s Voltage Law

• **Kirchhoff’s voltage law (KVL):** The algebraic sum of all voltages around a closed path (or loop) in a circuit equals zero.

\[
\sum_{m=1}^{N} v_m = 0
\]

- \(N\) = the number of voltages in the loop
- \(v_m\) = mth voltage
What is a Closed Path or Loop?

- A closed path or loop is a path that starts at an arbitrary selected node, and that traces a closed path in a circuit through selected basic circuit elements and return to the original node without passing through any intermediate node more than once.
Kirchhoff’s Voltage Law (cont.)

- To use Kirchhoff’s voltage law, an algebraic sign corresponding to a reference polarity must be assigned to every voltage in the path, e.g. a positive sign for a voltage rise requires a negative sign for a voltage drop.
A Single-Loop Circuit Illustrating KVL

\[ -v_1 + v_2 + v_3 - v_4 + v_5 = 0 \]

\[ v_2 + v_3 + v_5 = v_1 + v_4 \]

\textbf{Sum of voltage drops} = \textbf{Sum of voltage rises}
A Simple Application of KVL

Combine voltage sources in series.
(a) The original circuit.
(b) The equivalent circuit.

\[-V_{ab} + V_1 + V_2 - V_3 = 0\]

\[V_{ab} = V_1 + V_2 - V_3\]
Analysis of Resistor Circuits

• By Combining Ohm’s law, KCL, and KVL, we can analyze any resistor circuits, that is, a circuit with resistors and active sources.
Example 2.5

Q: Find $v_1$ and $v_2$

Ohm’s Law: $v_1 = 2i, \quad v_2 = -3i$

KVL: $-20 + v_1 - v_2 = 0$

$-20 + 2i + 3i = 0 \implies i = 4 \text{ A}$

$v_1 = 8 \text{ V}, \quad v_2 = -12 \text{ V}$.
Example 2.6

Q: Determine $v_0$ and $i$

\[ \text{KVL: } -12 + 4i + 2v_0 - 4 + 6i = 0 \]

\[ \text{Ohm’s Law: } v_0 = -6i \]

\[ -16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A} \]

\[ v_0 = 48 \text{ V}. \]
Example 2.7

Q: Determine $i_o$ and $v_o$

**KCL:** $3 + 0.5i_o = i_o \implies i_o = 6 \text{ A}$

**Ohm’s Law:** $v_o = 4i_o = 24 \text{ V}$
Example 2.8

Q: Find currents and voltages in the following circuit.
Example 2.8 (cont.)

Ohm's Law: \( v_1 = 8i_1, v_2 = 3i_2, v_3 = 6i_3 \)

KCL at node \( a \): \( i_1 - i_2 - i_3 = 0 \) \( \cdots \cdots (1) \)

KVL to Loop 1: \(-30 + v_1 + v_2 = 0\)
\(-30 + 8i_1 + 3i_2 = 0\)
\(\Rightarrow \)
\(i_1 = \frac{(30 - 3i_2)}{8} \) \( \cdots \cdots (2) \)

KVL to Loop 2: \(-v_2 + v_3 = 0\) \(\Rightarrow v_3 = v_2\)

\(6i_3 = 3i_2 \Rightarrow i_3 = \frac{i_2}{2} \) \( \cdots \cdots (3) \)

Combine (1)~(3)
\[ \frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0 \]
\(\Rightarrow i_2 = 2 \, \text{A} \)
\(i_1 = 3 \, \text{A}, \quad i_3 = 1 \, \text{A}, \quad v_1 = 24 \, \text{V}, \quad v_2 = 6 \, \text{V}, \quad v_3 = 6\text{V}\)
Resistors in Series

- Remember that two elements are connected in series when they connect at a single node.
- **Series-connected circuit elements** carry the same current. Prove by applying KCL to each node in the circuit.
- Focus on reducing complex circuits into simpler, *equivalent circuits*. 
A Single-loop Circuit with Two Resistors in Series

Ohm's Law: \( v_1 = iR_1, v_2 = iR_2 \)

KVL at the loop: \(-v + v_1 + v_2 = 0\) \(\Rightarrow\) \(i = \frac{v}{R_1 + R_2}\)
Equivalent Circuit of the Previous Circuit

Rewrite $v = i(R_1 + R_2)$ as $v = iR_{eq}$

• Hence, $R_1$ and $R_2$ can be replaced by an equivalent resistor $R_{eq}$; that is

$R_{eq} = R_1 + R_2$
• The equivalent resistance $R_{eq}$ of any number of resistors connected in series is the sum of the individual resistances, $R_i$’s. Note $R_{eq} > R_i$.

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$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$
The Voltage-Divider Circuit

- **Voltage-divider circuits** are used to develop more than one voltage level from a single voltage supply.

- **Principle of voltage division:** The source voltage \( v \) is divided among the resistors in series in direct proportion to their resistances.

- **Voltage division formula for resistors in series:**
  Given a set of resistors \( R_1, R_2, \ldots, R_N \), in series, with the source voltage \( v \), the \( n \)th resistors (\( R_n \)) will have a voltage drop of

\[
  v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v
\]
The series connected resistors can realize the voltage-divider circuits which develop more than one voltage level from a single voltage supply.

For example:

\[ v_1 = \frac{R_1}{R_1 + R_2} v, \quad v_2 = \frac{R_2}{R_1 + R_2} v \]
Resistors in Parallel

• Remember that two elements are connected in parallel when they connect at a single node pair.

• Parallel-connected circuit elements have the same voltage across their terminals.

• Focus on reducing complex circuits into simpler, equivalent circuits.
Two Resistors in Parallel

Ohm's Law: \( v = i_1 R_1 = i_2 R_2 \)

KCL at node \( a \): \( i = i_1 + i_2 \) \( \Rightarrow \) \( i = \frac{v}{R_1} + \frac{v}{R_2} = v \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \)
Equivalent Circuit of the Previous Circuit

Rewrite

\[ i = v \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

as

\[ i = \frac{v}{R_{eq}} \]

• Hence, \( R_1 \) and \( R_2 \) can be replaced by an equivalent resistor \( R_{eq} \); that is

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]

\[ \Rightarrow \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \]

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Equivalent Resistance of Resistors in Parallel

• The inverse of the equivalent resistance $1/R_{eq}$ is the sum of inverses of all $N$ resistors in parallel. Note $R_{eq} < R_i$

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$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$

If $R_1 = R_2 = \cdots = R_N = R$

$$R_{eq} = \frac{R}{N}$$
Equivalent Conductance of Resistors in Parallel

- Since the conductance is equal to the inverse of resistance, we have
- The equivalent conductance of resistors connected in parallel is the sum of each conductance.

\[
G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N
\]

where \( G_{eq} = 1/R_{eq}, \ G_1 = 1/R_1, \ G_2 = 1/R_2, \cdots G_N = 1/R_N. \)
The inverse of equivalent conductance of resistors connected in parallel is the sum of the inverses of each resistor’s conductance.

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{G_N}$$

where $G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_N = 1/R_N$. 
The Current Divider Circuit

- **Principle of current division:** The source current $i$ is shared among the resistors in parallel in inverse proportion to their resistances.

- **Current division formula for resistors in parallel:** Given a set of resistors $R_1, R_2, \ldots, R_N$, in parallel, with the source current $i$, the $n$th resistors ($R_n$) will have current

  $$i_n = \frac{1}{R_n} + \frac{1}{R_1} + \cdots + \frac{1}{R_N} \quad i = \frac{G_n}{G_1 + G_2 + \cdots + G_N}$$
The series connected resistors can realize the current-divider circuits which develop more than one current level from a single current source.

For example:

\[ i_1 = \frac{R_2}{R_1 + R_2} \cdot i = \frac{G_1}{G_1 + G_2} \cdot i \]
\[ i_2 = \frac{R_1}{R_1 + R_2} \cdot i = \frac{G_2}{G_1 + G_2} \cdot i \]
Two Extremely Cases

(a) A shorted circuit
   - \( R_{eq} = 0 \)
   - The entire current flows through the short circuit.

(b) An open circuit
   - \( R_{eq} = R_1 \)
   - The entire current flows through the least resistance.
Example 2.9

Q: Find $R_{eq}$ of the below circuit.

\[ 6 \Omega \| 3 \Omega = \frac{6 \times 3}{6 + 3} = 2 \Omega \]

\[ 1 \Omega + 5 \Omega = 6 \Omega \]
Example 2.9 (cont.)

In Fig. (a),

\[ 2\Omega + 2\Omega = 4\Omega, \]

\[ 4\Omega \parallel 6\Omega = \frac{4 \times 6}{4 + 6} = 2.4\Omega \]

In Fig. (b),

\[ R_{eq} = 4\Omega + 2.4\Omega + 8\Omega = 14.4\Omega. \]
Example 2.10

Q: Find $R_{ab}$ of the below circuit.

\[ R_{ab} = 3 \Omega \parallel 6 \Omega = \frac{3 \times 6}{3 + 6} = 2 \Omega; \quad 12 \Omega \parallel 4 \Omega = \frac{12 \times 4}{12 + 4} = 3 \Omega \]

\[ 1 \Omega + 5 \Omega = 6 \Omega. \]
Example 2.10 (cont.)

In Fig. (a)

\[ 1\Omega + 3\Omega \parallel 6\Omega = 1 + \frac{3 \times 6}{3 + 9} = 3\Omega \]

In Fig. (b)

\[ 2\Omega \parallel 3\Omega = \frac{2 \times 3}{2 + 3} = 1.2\Omega \]

\[ R_{ab} = 10 + 1.2 = 11.2\Omega \]
Example 2.11

Q: Find the equivalent conductance $G_{eq}$ of circuit (a).

Method 1:

In Fig. (a)

$8 \, \text{S} + 12 \, \text{S} = 20 \, \text{S}$

In Fig. (b)

$\frac{20 \times 5}{20 + 5} = 4 \, \text{S}$

$G_{eq} = 6 + 4 = 10 \, \text{S}$
Method 2:

\[ R_{eq} = \frac{1}{6} \left( \frac{1}{5} + \frac{1}{8} \parallel \frac{1}{12} \right) = \frac{1}{6} \left( \frac{1}{5} + \frac{1}{20} \right) = \frac{1}{6} \frac{1}{4} \]

\[ = \frac{1}{6} \times \frac{1}{4} = \frac{1}{10} \Omega \]

\[ G_{eq} = \frac{1}{R_{eq}} = 10 \text{ S.} \]
Example 2.12

Q: Find $i_0$ and $v_0$, and calculate the power dissipated in 3-$\Omega$ resistor of circuit (a).

In Fig. (a): $6\Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\Omega$

$$i = \frac{12}{4 + 2} = 2 \text{ A}$$

In Fig. (b): $v_0 = \frac{2}{2 + 4} (12 \text{ V}) = 4 \text{ V}$

分壓公式

$$v_0 = 3i_0 = 4 \implies i_0 = \frac{4}{3} \text{ A}$$

分流公式

$$i_0 = \frac{6}{6 + 3} i = \frac{2}{3} (2 \text{ A}) = \frac{4}{3} \text{ A}$$

$$p_0 = v_0 i_0 = 4 \left( \frac{4}{3} \right) = 5.333 \text{ W}.$$
Example 2.13

Q: Find (a) $v_0$, (b) the power supplied by the current source, (c) the power absorbed by each resistor.

(a) $6 + 12 = 18k\Omega$

$$i_1 = \frac{18000}{9000 + 18000} (30 \text{ mA}) = 20 \text{ mA}$$

$$i_2 = \frac{9000}{9000 + 18000} (30 \text{ mA}) = 10 \text{ mA}$$

$$v_0 = 9000i_1 = 18000i_2 = 180 \text{ V}.$$  

(b) $p_0 = v_0i_0 = 180(30) \text{ mW} = 5.4 \text{ W}.$
Example 2.13 (cont.)

(c) 12-kΩ resistor

\[ p_{12} = i_2^2 R_{12} = i_2^2 R_{12} = (10 \times 10^{-3})^2 (12000) = 1.2 \text{ W} \]

6-kΩ resistor

\[ p_6 = i_6^2 R_6 = i_2^2 R_6 = (10 \times 10^{-3})^2 (6000) = 0.6 \text{ W} \]

9-kΩ resistor

\[ p_9 = \frac{v_9^2}{R_9} = \frac{v_0^2}{R_9} = \frac{(180)^2}{9000} = 3.6 \text{ W} \]

or

\[ p_9 = v_9 i_9 = v_0 i_1 = 180(20) \text{ mW} = 3.6 \text{ W} \]

The power supplied 5.4 W = 1.2 W + 0.6 W + 3.6 W.
The Bridge Network

How do we combine resistors $R_1$ through $R_6$ when they are neither series nor parallel?
Delta (Δ)-to-Wye (Y) Equivalent Circuits

• Some circuits of interconnected resistors *cannot* be reduced to an equivalent resistance using the simple series of parallel equivalent circuits introduced earlier.

• Delta (Δ) and pi (Π) circuits are electrically equivalent.

• Wye (Y) and tee (T) circuits are electrically equivalent.
Two forms of the same network: (a) Y, (b) T.
Two forms of the same network: (a) $\Delta$, (b) $\Pi$. 

(a)

(b)
Delta ($\Delta$)-to-Wye (Y) Equivalent Circuits (cont.)

- For each pair of terminals in $\Delta$- and Y-connected circuits, the equivalent resistance can be got using series and parallel simplifications:

$$R_{12}(Y) = R_1 + R_3,$$

$$R_{12}(\Delta) = R_b \| (R_a + R_c) \implies$$

By $R_{12}(Y) = R_{12}(\Delta)$

$$R_{12} = R_1 + R_3 = \frac{R_b (R_a + R_c)}{R_a + R_b + R_c} \cdots (a)$$
Delta (Δ)-to-Wye (Y) Equivalent Circuits (cont.)

\[ R_{12} = R_1 + R_3 = \frac{R_b (R_a + R_c)}{R_a + R_b + R_c} \quad \cdots (a) \]

\[ R_{13} = R_1 + R_2 = \frac{R_c (R_b + R_a)}{R_a + R_b + R_c} \quad \cdots (b) \]

\[ R_{34} = R_2 + R_3 = \frac{R_a (R_b + R_c)}{R_a + R_b + R_c} \quad \cdots (c) \]

Eq. (a) – Eq. (c) \[ R_1 - R_2 = \frac{R_c (R_b - R_a)}{R_a + R_b + R_c} \]
Delta ($\Delta$)-to-Wye (Y) Equivalent Circuits (cont.)

- Y-connected resistors in terms of $\Delta$-connected resistors required for the $\Delta$-to-Y equivalent circuit:

\[
R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad (1)
\]

\[
R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad (2)
\]

\[
R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \quad (3)
\]
• Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

\[ R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \]
Superposition of $Y$ and $\Delta$ networks as an aid in transforming one to the other.

注意相對位置！
Wye (Y)-to-Delta (Δ) Equivalent Circuits

• Δ-connected resistors in terms of Y-connected resistors required for the Y-to-Δ equivalent circuit:

• By equations (1)~(3), we have

\[ R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_a R_b R_c (R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} \]

\[ = \frac{R_a R_b R_c}{R_a + R_b + R_c} \]  

(A)
Wye (Y)-to-Delta (Δ) Equivalent Circuits (cont.)

\[
R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \\
R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \\
R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}
\]

(A)/(1) 
(A)/(2) 
(A)/(3)

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.
Wye (Y)-to-Delta (Δ) Equivalent Circuits (cont.)

注意相對位置!

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$
A Special Case

\[ R_1 = R_2 = R_3 = R_Y, \quad R_a = R_b = R_c = R_\Delta \]

\[ R_Y = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_Y \]
An Example of $\Delta$ to $Y$

(a) \[ R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = 5 \, \Omega \]

(b) \[ R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5 \, \Omega \]

\[ R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3 \, \Omega \]
Example 2.15

Q: Obtain the equivalent $R_{ab}$ and use it to find the current $i$.

1. Define: the problem is clearly defined.

2. Present:
   a) remove voltage source
   b) a resistive circuit
   c) $\text{Y-}\Delta$ or $\Delta - \text{Y}$

3. Alternative: many different approaches can be used
Example 2.15 (cont.)

- The circuit contains 2 Y networks and 3 Δ networks.
- Transforming just one can simplify the analysis.
- We transform a Y to a Δ.

\[
R_1 = 10 \, \Omega, \quad R_2 = 20 \, \Omega, \quad R_3 = 5 \, \Omega
\]

\[
R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{350}{10} = 35 \, \Omega
\]

\[
R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{350}{20} = 17.5 \, \Omega
\]

\[
R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{350}{5} = 70 \, \Omega
\]
Example 2.15 (cont.)

- With the Y converted to Δ, combining the three pairs of resistors in parallel,

\[
70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21 \, \Omega
\]

\[
12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292 \, \Omega
\]

\[
15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5 \, \Omega
\]
Example 2.15 (cont.)

\[
R_{ab} = (7.292 + 10.5) \| 21 = \frac{17.792 \times 21}{17.792 + 21} = 9.632 \ \Omega
\]

Then

\[
i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458 \ \text{A.}
\]
Example 2.15 (cont.)

5. Evaluate (Solve it by Δ-Y transformation).

Let $R_c = 10 \, \Omega$, $R_a = 5 \, \Omega$, $R_n = 12.5 \, \Omega$

\[
R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545 \, \Omega
\]

\[
R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273 \, \Omega
\]

\[
R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182 \, \Omega
\]
Example 2.15 (cont.)

- Looking at the resistance between $d$ and $b$,

\[
R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642 \, \Omega.
\]

\[
R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = 9.631 \, \Omega.
\]

\[
i = \frac{V_s}{R_{ab}} = \frac{120}{9.631} = 12.46 \, \text{A}.
\]

6. Satisfactory?
Homework (Due day:)

- Problems 2.3, 2.4, 2.6, 2.13, 2.14, 2.25, 2.29, 2.38, 2.47, 2.53, and 2.57.